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Quasi-Steady Models for Dynamic Salt-Fresh Interface Analysis

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QUASI-STEADY MODELS FOR DYNAMIC SALT-FRESH INTERFACE ANALYSES

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Synopsis

Two quasi-steady methods for unsteady analyses of the motion of the salt-fresh interface in unconfined coastal aquifers are investigated. Their performance is evaluated through comparisons of the results from the quasi-steady models with results from a more exact model. The comparisons show that the quasi-steady models overestimate the rate of response of the interface. It is concluded that the more exact model should be used for one-directional flow studies despite the extra effort required. Quasi-steady models do have a place in two-dimensional analyses but their limitations, demonstrated by this study, should be recognised.

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1. INTRODUCTION

Predictions of the dynamic response of the salt-fresh interface are sometimes needed for planning the management of an unconfined coastal aquifer where salt water intrusion is a potential problem. In practical applications, it is often desirable that a simple, effective method be available for calculations. Some simple methods are based on a quasi-steady assumption. The purpose of the study described in this report was an assessment of the validty of results from quasi-steady models by comparisons with results from a more exact model.

2. THEORY

2.1 Problem Definition

The mathematical problem for which a solution if sought is that of a one-dimensional groundwater flow as shown in Figure 1. It is assumed that the Dupuit approximation applies and that there is a well defined interface between the salt and fresh water zones. The aquifer geometry and properties, $q^f(L,t)$ and $R(x,t)$ are assumed to be known. The unknowns for which a solution is sought are $s(x,t)$ and $\zeta(x,t)$. See Appendix A for definitions of terms.

2.2 Governing Equations

The governing equations (Shamir and Dagan (1971), Bear (1972)) are:

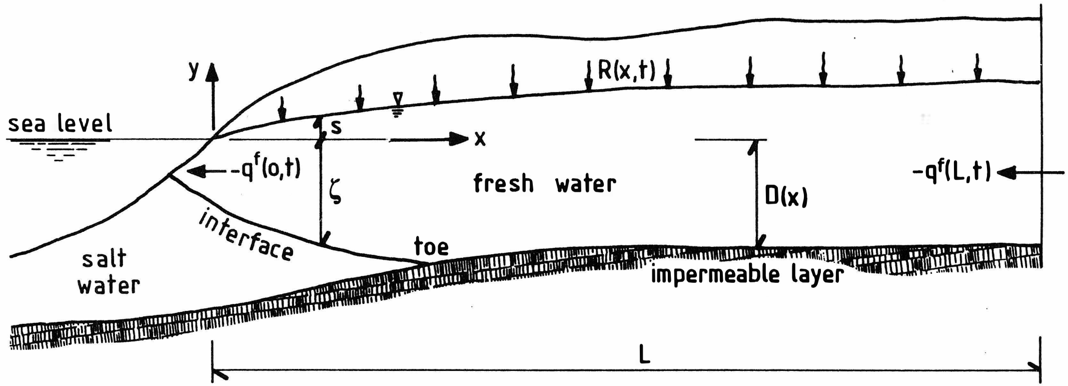

FIGURE 1: Vertical Section Through an Unconfined Coastal Aquifer

in freshwater zone for $x \leq x_t$

$$n^f \frac{\partial s}{\partial t} + n^s \frac{\partial \zeta}{\partial t} - \frac{\partial}{\partial x}\left[K^f(\zeta + s)\frac{\partial s}{\partial x}\right] = R \quad (1)$$

in saltwater zone for $x \leq x_t$

$$n^s \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[K^s(D - \zeta)\frac{\partial}{\partial x}\left(\frac{\gamma^f}{\gamma^s} s - \frac{\Delta\gamma}{\gamma^s}\zeta\right)\right] = 0 \quad (2)$$

and in freshwater zone for $x \geq x_t$

$$n^f \frac{\partial s}{\partial t} - \frac{\partial}{\partial x}\left[K^f(D + s)\frac{\partial s}{\partial x}\right] = R \quad (3)$$

2.3 Boundary Conditions

The boundary conditions that must be specified are

(i) $s(0,t)$ and $\zeta(0,t)$

(ii) $s(x,0)$ and $\zeta(x,0)$

(iii) either $q^f(L,t)$ or $s(L,t)$.

If the interface is shallow, the approximation $s(0,t) = 0$ and $\zeta(0,t) = 0$ for all t can be used without significant error (Shamir and Dagan (1971)). An alternative is the adoption of the steady state result

$$\zeta(0,t) = -f(\theta)\, q^f(0,t)\, \beta/K \quad (4)$$

where $f(\theta)$ depends on the slope, θ, of the ground surface where the aquifer meets the sea. $f(\theta) = 1$ for small slopes and $f(\theta) = 0.722$ for the vertical case (Shamir and Dagan (1971), Vappicha and Nagaraja (1976)).

The initial conditions $s(x,0)$ and $\zeta(x,0)$ must be known. For the purposes of this study, the assumption is made that steady conditions prevail for $t < 0$.

2.4 Steady State Conditions

Under steady conditions, Equation 2 shows that $\zeta = \beta\ s$ and the other governing equations become

$$- \frac{\partial}{\partial x}\left[K^f\ \alpha s\ \frac{\partial s}{\partial x}\right] = R, \quad 0 \leq x \leq x_t \tag{5a}$$

and

$$- \frac{\partial}{\partial x}\left[K^f (D + s)\ \frac{\partial s}{\partial x}\right] = R, \quad x_t \leq x \leq L \tag{5b}$$

If K and R are constant, the solution for $0 \leq x \leq x_t$ is

$$s^2 = - \frac{Rx^2}{\alpha K} - \frac{2q_o x}{\alpha K} + s_o^{\ 2} \tag{6}$$

in which

$$q_o = - K\left(\frac{\partial s}{\partial x}\right)_o \alpha s_o = q^f(L,0) - RL \tag{7}$$

The toe is located at the position where $s = D/\beta$. Therefore,

$$x_t = - \frac{q_o}{R} - \left\{\left(\frac{q_o}{R}\right)^2 - \left[\left(\frac{D}{\beta}\right)^2 - s_o^{\ 2}\right]\frac{\alpha K}{R}\right\}^{1/2} \tag{8}$$

for $R \neq 0$

or

$$x_t = \frac{-\alpha K\left[\left(\frac{D}{\beta}\right)^2 - s_o{}^2\right]}{2q_o} \qquad (9)$$

for R = 0

If D is constant, the solution for $x_t \leq x \leq L$ is

$$(D + s)^2 = \frac{Rx^2}{K} + \frac{2}{K}\left(RL - q^f(L,0)\right) x + C \qquad (10)$$

The value for C is determined from the condition that $s = D/\beta$ at $x = x_t$.

3. METHODS OF SOLUTION

3.1 Shamir and Dagan's Solution

A general analytical solution of Equations 1, 2 and 3 is not currently possible. Shamir and Dagan (1971) have published a numerical, finite difference method (referred to as Method A) which uses different grid spacings for $x \leq x_t$ and $x \geq x_t$. s and ζ are evaluated for $x < x_t$ and s is evaluated for $x > x_t$ for each time step. The new toe position is calculated. If the toe movement is greater than the grid spacing for $0 \leq x \leq x_t$, the time interval is reduced and the calculations are repeated. When the calculated toe movement is acceptable, a new grid is generated for the next time step. Shamir and Dagan's method is probably the best available for obtaining general solutions and has been adopted as the basis for evaluation of the other methods. However, it has some disadvantages:

(i) it is relatively difficult to program and the degree of difficulty would increase substantially for problems with two dimensional flows,

(ii) it is not suited to a numerical analysis in which a fixed grid is desirable, and

(iii) because it is a numerical method, it does not yield general solutions. (However, the generality of any one solution can be improved if the analysis is done for a non-dimensional problem).

If the problem can be simplified through the assumption of a quasi-steady condition, the advantages include:

(i) approximate, analytical methods may be used to obtain general solutions for some problems, and

(ii) programming effort and computational times for numerical solutions are significantly reduced and fixed grids may be used. This is important for two-dimensional, regional models of coastal aquifers.

Two approximate methods based on the assumption of a quasi-steady condition have been studied by a comparison of the results obtained with the results from Shamir and Dagan's method. The first is an approximate, analytical method developed by Vappicha and Nagaraja (1976) (called Method B) and the second is a numerical finite difference method (which uses a fixed grid and eliminates ζ) developed by the author (Method C).

3.2 Vappicha and Nagaraja's Method

Vappicha and Nagaraja's solutions are restricted to cases where $R = 0$ and neglect any changes in fresh water storage due to changes in the water table. They assumed that the interface at any time, t, conformed to the steady state profile determined by the value of $q^f(0,t)$. A weakness in their presentation is that their equations are written terms of $\zeta(0,t)$ and, therefore, $\zeta(0,t)$ *appears* to be the important, independent parameter of their solutions. This is not consistent with the fact that, if the interface is shallow, the approximation $\zeta(0,t) = 0$ is acceptable. Although Vappicha and Nagaraja base their solutions on a differential equation for $d(\zeta(0,t))/dt$, $\zeta(0,t)$ is determined from $q^f(0,t)$ through Equation 4 and $q^f(0,t)$ is the fundamental variable on which their solutions are based as the following presentation shows.

The volume of water in the salt wedge for $0 \leq x \leq x_t$ is

$$(\mathrm{Vol})^s = n \int_{\zeta}^{D} y \, dx \tag{11}$$

If the assumption is made that, for a given aquifer, $\zeta(x,t)$ is a function of $q(0,t) \equiv q_o$, then

$$(\mathrm{Vol})^s = f(q_o) \tag{12}$$

and

$$\frac{d(\mathrm{Vol})^s}{dt} = \frac{df}{dt} = \frac{df}{dq_o}\frac{dq_o}{dt} \tag{13}$$

If the volume of the salt wedge increases, freshwater is displaced by the moving interface and continuity of the control volume of fresh water between $0 \leq x \leq x_t$ requires

$$\frac{d(\mathrm{Vol})^s}{dt} = q^f(x_t,t) - q^f(0,t) \tag{14}$$

Hence

$$\frac{dq_o}{dt} = \frac{q^f(x_t,t) - q_o}{df/dq_o} \tag{15}$$

If Equation 15 is solved for $q_o \equiv q(0,t)$, $\zeta(x,t)$ can then be evaluated.

In general Equation 15 is not amenable to an exact solution and Vappicha and Nagaraja used a fourth-order Runge-Kutta method to obtain numerical solutions. However, for the special case when $q^f(x_t,t)$ = constant for $t > 0$, an exact solution is given by Vappicha and Nagaraja. This case has been adopted for the comparisons done in this study.

3.3 The Author's Method

This method is based on the assumption that the steady condition, $\zeta = \beta\ s$ can be used in the unsteady analysis. Therefore, $\zeta(x,t) = \beta\ s(x,t)$ replaces Equation 2 and ζ can be eliminated from Equation 1. Equations 1 and 3 are solved for $s(x,t)$ using a finite-difference method based on a fixed grid. The toe position is determined from the condition that $\beta\ s(x,t) = D$ at x_t. The method was developed for applications where the approximation $s(0,t) = 0$ can be adopted.

4. TEST PROBLEMS

4.1 Test 1

The date for Test 1 were (see Figure 1):

$L = 1000$ m

$n = 0.2$

$R(x,t) = 0$

$q^f(L,t) = q^f(0,t) = -20$ m²/day, $\quad t < 0$

$q^f(L,t) = -10$ m²/day, $\quad t > 0$

$D(x) = 100$ m

$K = 60$ m/day

Because Vappicha and Nagaraja neglect storage due to water table movement, $q^f(x_t,t)$ equals $q^f(L,t)$ when their solution is used.

The toe movements following the instantaneous change in fresh water discharge, $q^f(L,t)$, were computed by each of the three methods and the results are shown in Figure 2.

The initial toe position was $x_t(0) \doteqdot 384$ m. The final position, $t \to \infty$, under steady conditions for $q^f(L,t) = -10\ m^2/day$ would be $x_t(\infty) \doteqdot 767$ m. The toe would move 383 m from its initial position to the new equilibrium position. A summary of the computed movements in 500 days is given in Table 1.

TABLE 1. Toe Movements in 500 days Test 1

Method (see text)	x_t (m) (t = 500 days)	Δx_t (m)	$\frac{\Delta x_t}{383}$
A	560	176	0.46
B	665	281	0.73
C	710	326	0.85

$$\Delta x_t = x_t(500) - x_t(0).$$

Both quasi-steady methods predict a significantly faster response of the interface than does Method A. The author's method produced the largest errors.

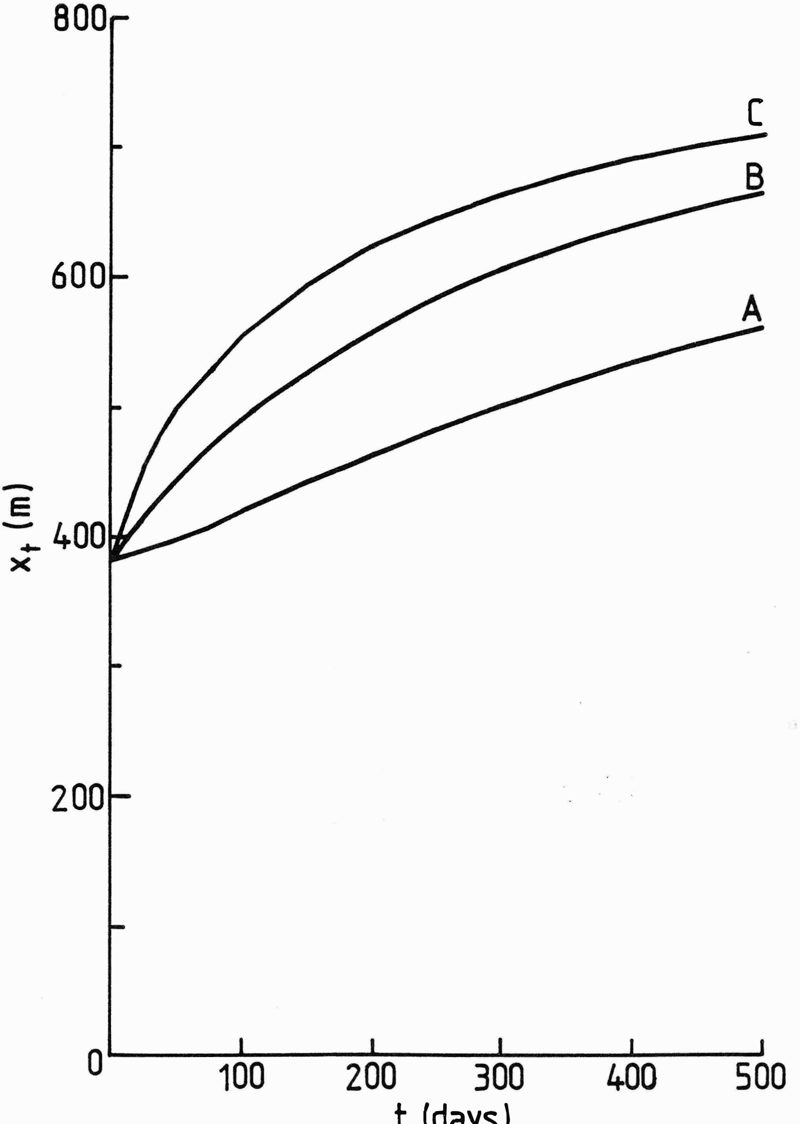

FIGURE 2: Toe Movements for Test 1

4.2 Test 2

The choice for K of 60 m/day resulted in a relatively shallow interface, In Test 2 a value for K of 10 m/day was adopted which resulted in a relatively steep interface.

Since Method B neglects storage due to water table movements, some of the differences between the results from this method and from Method A can be attributed to this storage which depends on the value of L. The effect of varying L in analyses by Method A was included in this test. Values of L used were 1000 m, 500 m, 200 m (Runs A1, A2, A3).

The author's model is not well suited to this application. However, it was modified so that the value of $s(0,t) = \zeta(0,t)/\beta$ with $\zeta(0,t)$ calculated from Equation 4 with $q^f(0,t) = q^f(L,t)$. A value for L of 250 m was used.

The toe movements computed in the various analyses are shown in Figure 3.

The initial toe position, $x_t(0)$, was 42.7 m and the final steady state position, $x_t(\infty)$, would be 117.4 m. The quasi-steady solutions indicate the movement of the interface is virtually completed at the end of a 500 day period. Table 2 shows the times computed for the toe to move to x_t = 60, 80, 100 m (approximately 0.25, 0.50, 0.75 of the total movement to the new equilibrium position).

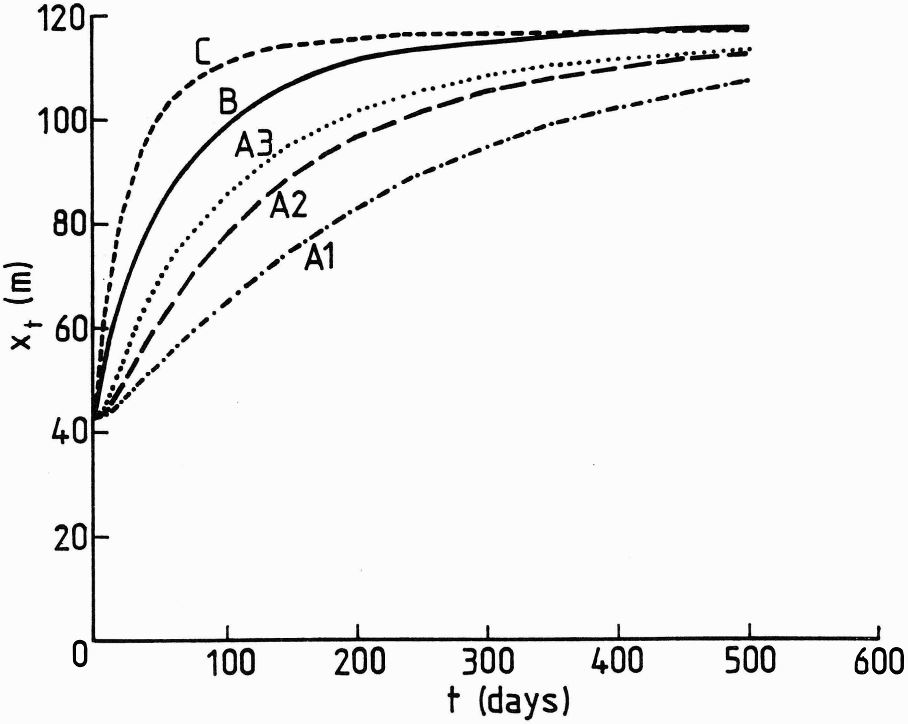

FIGURE 3: Teo Movements for Test 2

TABLE 2. Times for specified toe movements Test 2

Method	Times (days) for toe movements to		
	x_t = 60 m	x_t = 80 m	x_t = 100 m
A1	78	184	368
A2	45	106	236
A3	31	80	190
B	14	44	108
C	7	19	48

As in Test 1, both quasi-steady methods significantly overestimate the rate of response of the interface and the author's method gives the worst results. The effect of storage due to water table movement is important and the agreement between the results from Method B and the results from Method A improves as L decreases but the differences are still significant.

Some insight into the differences can be gained from consideration of the velocity, $V_t = dx_t/dt$, at which the toe moves.

Figure 4 shows the computed velocities from Methods A and B and Figure 5 shows the results from Method C for Test 2. There are reasons for believing that the initial response of the interface will be very slow and that the rate of movement will be a maximum some time after a change in conditions (Isaacs 1983). The results from Method A confirm these conclusions. The quasi-steady model of Vappicha and Nagaraja cannot reproduce this behaviour and predicts a maximum rate of movement at the instant the change occurs. Because Method B predicts an instantaneous response by the interface to a change in conditions, the results from this model may be in error not only

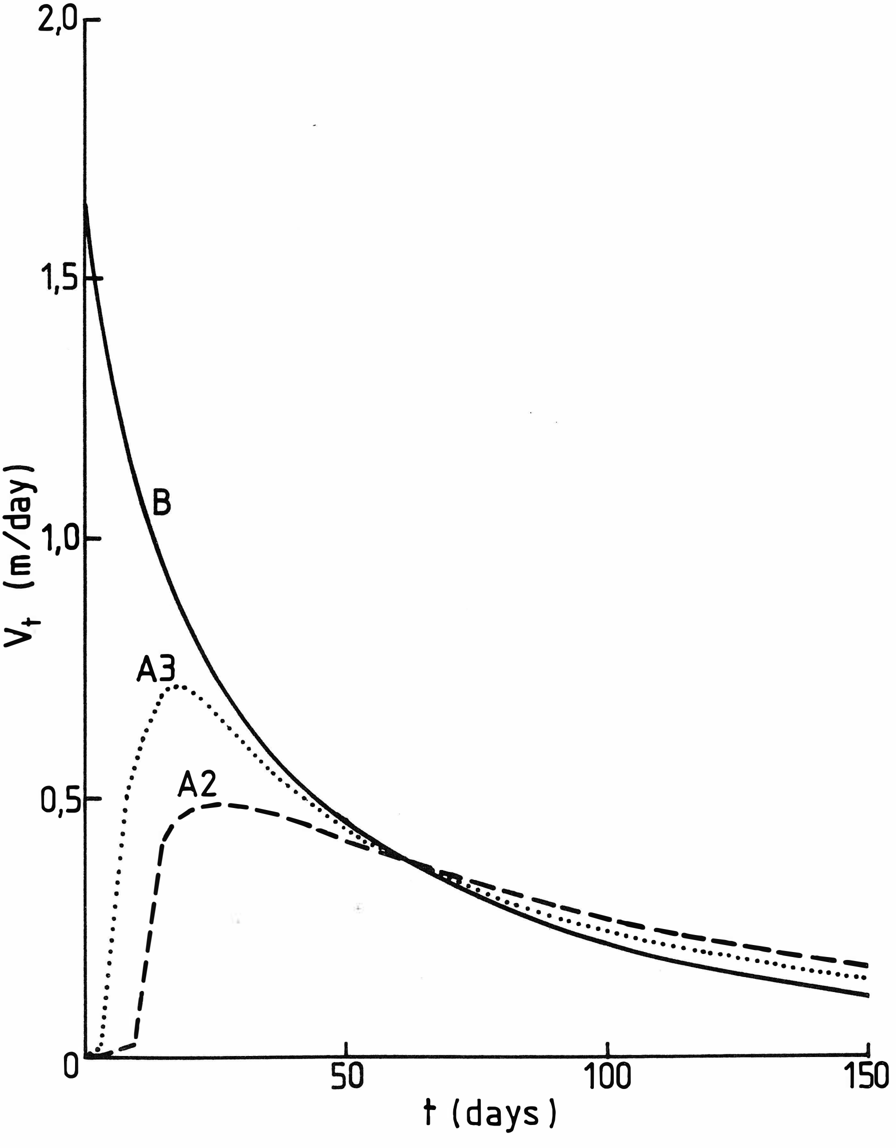

FIGURE 4: Velocity of Toe Movement - Test 2
Methods A, B

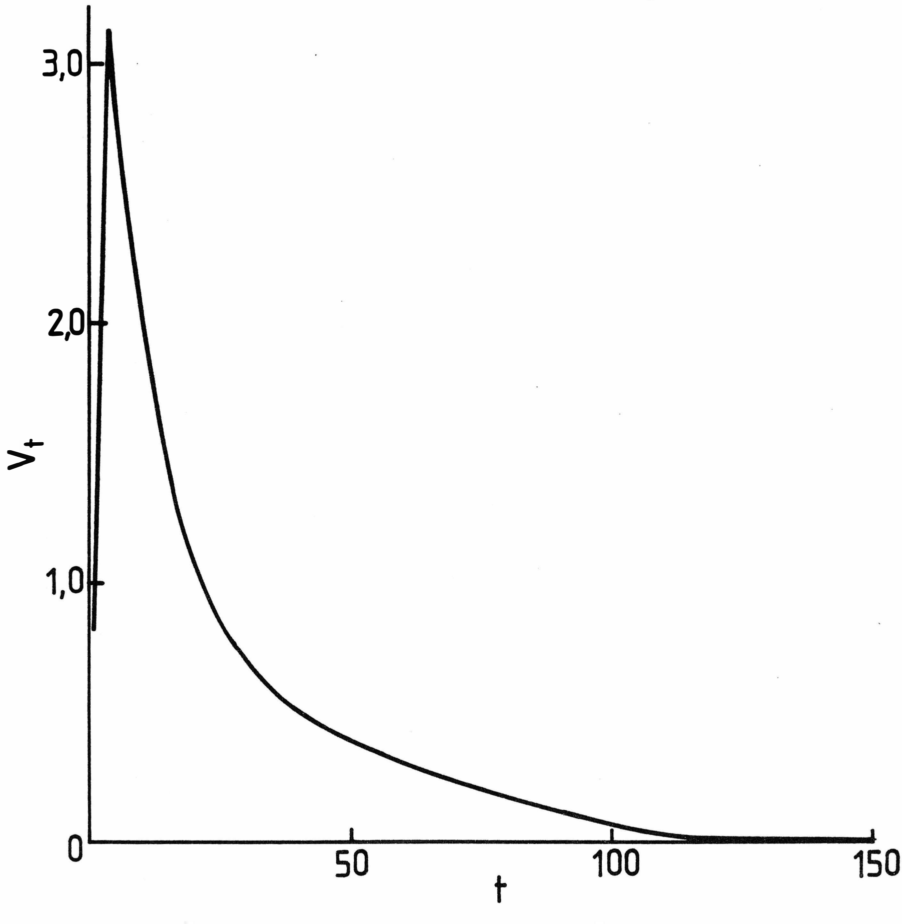

FIGURE 5: Velocity of Teo Movement - Test 2
Method C

in predicting the toe position but also in predicting the direction of motion at a particular time. For example, if the interface were moving inland when a large negative increase in $q^f(L,t)$ occurred, it would continue to do so for some time after the change. Method B would predict a change in direction at the time of the increase in $-q^f(L,t)$.

Method C not only produces a rapid response to a change but grossly overestimates the early rate of response of the interface. The maximum toe velocity computed was approximately four times that from the analyses with Method A. A major reason for these large initial rates was the use of the assumption that $q^t(0,t)$ equalled $q^t(L,t)$ in the calculation of $s(0,t)$. A preliminary assessment of the effect of a more reasonable assumption for $q^f(0,t)$ indicated that the maximum toe velocity might be halved. However, given that the results from Test 1 are not significantly affected by the assumption adopted for $s(0,t)$ and that Method C performed poorly, further studies on Method C do not appear warranted.

5. CONCLUSION

The objective of this study was the assessment of two quasi-steady methods for predicting the dynamic response of the salt-fresh interface through comparisons with the method of Shamir and Dagan. The results of the tests show that:

(i) the quasi-steady models overestimate the rate at which the interface responds to a change in fresh water discharge,

(ii) a deficiency in the quasi-steady models is their inability to model the initial lag in the interface response,

(iii) as a result of (i) and (ii) the quasi-steady models may have significant errors in their estimates of the interface position and may even be wrong in their predictions of the direction of movement at a particular time, and

(iv) the storage due to movement of the water table may have a significant effect on the rate at which the interface responds to a change in fresh water flow.

For unsteady analyses of interface motion with one-directional groundwater flow, the advantages derived from a quasi-steady assumption are outweighed by the loss of accuracy in the results. The extra effort in programming the method of Shamir and Dagan is worthwhile.

For unsteady regional groundwater models of coastal aquifers (two-dimensional flows) the appropriate level of modelling depends on the purpose of the model and the importance of the predictions of the unsteady interface position. The simplest models neglect the existence of the salt water wedge and, for many analyses, this is acceptable. A very sophisticated model could include the effects of the salt-fresh interface modelled by the method of Shamir and Dagan but the programming effort required would be great. Intermediate models which use a quasi-steady assumption clearly have a role between the two extremes but their limitations should be recognised.

APPENDIX A - NOMENCLATURE

Symbol	Meaning
$D = D(x)$	thickness of aquifer from sea level down to impervious layer
f	(as superscript) refers to fresh zone
g	acceleration due to gravity
$h = \frac{p}{\rho g} + y$	piezometric head
K	hydraulic conductivity
L	length of aquifer
n^f	effective porosity for movement of the phreatic surface
n^s	effective porosity for movement of the interface
p	pore water pressure
$q = q(x,t)$	two dimensional seepage discharge
$R = R(x,t)$	nett discharge into aquifer from above
$s = s(x,t)$	elevation of phreatic surface above sea level
s	(as superscript) refers to salt zone
t	time
$T = T(x,t)$	transmissivity
V_t	velocity at which toe moves
v	Darcy velocity
x	horizontal distance measured from origin at intersection of sea level and land surface
$x_t = x_t(t)$	distance to toe of interface
y	vertical distance measured from origin
α =	$1 + \beta$
β =	$(\gamma^s - \gamma^f)/\gamma^f$
ρ	density of water
γ	specific weight of water
$\Delta\gamma$ =	$\gamma^s - \gamma^f$
$\zeta = \zeta(x,t)$	distance from sea level to the interface

APPENDIX B - REFERENCES

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